Modeling Secondary Services in Engineering and Mapping

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***Abstract*—With the exponential growth in computer hardware power and computer software functionality, it is now possible to include distribution transformers and secondary systems in our engineering models. With the ever growing demands and expectations associated with "Smart Grid," it is further becoming a requirement that we model secondary. The traditional engineering model has ended at the high side of the distribution transformer and did not necessarily include every foot of distribution line. Secondary and distribution transformers add directly to the size and complexity of the engineering database required to model each transformer and all secondary to each meter. They also add to the size and complexity of the primary system because almost all, if not all, of the primary must be modeled in order to provide connection points for transformers. This paper will discuss the reasons for modeling secondary, define the modeling requirements of center-tapped transformers, define the modeling requirements of secondary and services, explore the sensitivity of data accuracy when modeling secondary systems, explain the derivations of X/R for center-tapped transformers, illustrate transformer X/R calculations when supplied with transformer loss data, and summarize the algorithm theory required to accurately model center-tapped transformers and secondary triplex systems.**

***Index Terms*—ANSI, distribution systems, impedance matrices, modeling, reactance, resistance, secondary services, sensitivity analysis, Smart Grid, theory, transformers, voltage drop.**

I. Introduction

THE DISTRIBUTION engineer must ensure the power system is designed and operated within ANSI voltage limits as well as criterion or guidelines developed within the utility that may be even more stringent. Planning for and constructing the system in the most economical and reliable way such that every customer attains sufficient capacity and suitable voltage is of utmost importance. The primary voltage guidelines of the utility must take into account voltage drop through the transformer and secondary service conductor, which typically may be anywhere between 1 and 4 V. As a result, this variance on the secondary service drops might compel planners to only address the primary voltage levels when analyzing and planning systems. However, today's robust computing power coupled with center-tapped transformer and triplex derivations applied to engineering software creates the capability of a full system study with a completely detailed model down to the consumer level if desired.

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II. Evolution of Computer Software and Hardware

Modeling the electric distribution system in detail from the substation to each customer meter was not possible until just the last few years. Except for the inaccessible and expensive "Super Computers," we simply did not have the computing power required. Beginning in the late 1980s, introduction of the "Personal Computer" started an evolutionary process that has now resulted in laptops with more computing power than the best "Mainframes" of the 70s and 80s and Server Networks with more computing power than the "Super Computers" of this earlier time.

This evolution makes it possible to model the electric circuit model in detail, and modeling in detail is necessary if we are to actually implement useful, efficient, and beneficial "Smart Grids" at the distribution level. Modeling secondary is not easy, and it will take time, effort, and investment to accomplish. This will lead many utilities to take shortcuts and to try to implement Smart Grid applications without spending the time, effort, and dollars required. In some cases, this might be the appropriate decision, but as our computer resources continue to grow in power and capability, it is just a matter of time before all successful utilities have made this important data step.

III. Geographic Information Systems (GIS) and Engineering Analysis (EA) Perspectives

To effectively perform analysis on secondary voltage portions of power systems, utilities must first have in place the data necessary to obtain accurate results. There are several resources that can be used to compile the data necessary for secondary analysis. The first place to start is to evaluate the existing data that may reside in the mapping or GIS system. Although the data in the system may create a visually appealing map, the real data needed for analysis go far beyond a pretty picture of the electrical system. Other data sources, for example, a transformer database from a warehouse or purchase agent, may provide some data for specific equipment that is installed on

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the system. Below is an outline of data recommendations and requirements when performing analyses on secondary systems.

A. Transformers

Transformers are typically modeled as bank objects in the GIS with individual units assigned to the corresponding phases of the bank. Some of the information noted below will be at the bank level, while other information will need to be associated with the unit.

Data necessary for the transformer include, but are not limited to, kVA, percent impedance, X/R, no-load losses, winding connection, primary connection, and input/output voltages.

The kVA, percent impedance, X/R, and no-load losses are all attributes of the transformer units that are installed at a location. These values will vary even between transformers of the same size. When collecting the data, it is up to the utility to decide if they wish to generalize these values and setup and average for all transformers of the same size or if they would like to maintain the correct values for each individual transformer unit on the system. Of course, by generalizing values, a level of accuracy is being sacrificed by not having the true values at each location. It may be easy to get this data for new transformer units, but for transformers that may have been installed for decades, this data may not exist. If this is the case, it may be necessary to collect all data possible for some transformers, and for transformers where the data cannot be collected, creating a set of average values for each size may be appropriate in certain cases. Some of these values, like X/R, can be calculated given other data for the transformer. This is covered later in this paper.

Many GIS systems will have a field called "Output Voltage" which allows the user to store the output voltage of the transformer. Typically, the data stored in this field may be collected as something like 120/240. The problem with this type of collection is that 120/240 is not a number. Since the goal is to perform analysis on the model, the Engineering Analysis system is going to require that values like voltage be stored as numbers. Hence, if possible, it will be best to modify the GIS data structure to allow for separate fields for each piece of data collected, for example a L-L Output stored as 240 and a L-G Output stored as 120.

The winding connection is used for determining how the transformer units are configured on the system. The winding connections available will vary depending on your Engineering Analysis software, so the provider needs to be contacted to see what options are available and what should be stored in the GIS.

Another piece of necessary data is the phasing of the center-tapped connected transformer. When modeling multiple transformers in a bank configuration, the transformer may be set up so one of the units provides 120/240-V single-phase service, while the bank also provides three-phase service to another load. Knowing which unit is providing the single-phase service will properly allocate loads to the correct transformers, resulting in more accurate loading and analysis.

Connectivity is another important attribute. It is important that either the transformers upline parent element is maintained in the GIS database, or that there is a way for the transformer and line to be evaluated to know that they are indeed connected to one another.

B. Secondary/Service Wire

The data required for secondary and service wire does not differ much from that of primary service wire. The data required for analysis are typically a wire size, wire configuration, transformer connection, phase, and length.

The wire size will typically contain both a size and a type, 1/0 AAC for example. The configuration would consist of a representation of the way the wires are constructed (Duplex, Triplex, and Quadriplex). It is not necessary to combine these fields and represent the service wire as 1/0 AAAC Triplex. The triplex designation would be used as the construction of the line, and the standard 1/0 AAAC definition would contain the attributes of that wire type and size. Maintaining the 1/0 AAAC Triplex could result in the user having to maintain multiple instances of the same wire and having to maintain redundant data. The only variable that will be different between a secondary wire and a primary wire of the same size will be the ampacity, so be sure this value is properly populated so overloading on the wire is properly calculated.

The construction definition would basically be the orientation of the wires. For a typical Triplex, there would be two insulated wires and one twisted bare wire. Simply storing the word Triplex as the construction allows the Engineering Analysis system to maintain a random spacing characteristic for the wires and properly calculate the impedance matrix.

The transformer connection value is the transformer bank in which the wire is connected. This maintains connectivity in the database so there is no question about which wire is connected to which transformer. The phasing designation of the wire will determine which transformer unit is the parent element of the secondary wire on transformer bank objects. If there are several secondary or service wire sections in series, connectivity between the wire sections will also need to be maintained. Again, it is not necessary to store an explicit connectivity path for the elements, but there must be some way to evaluate the data to determine what the connectivity should be.

C. Service Locations

One of the most important parts of maintaining a secondary model is the representation of the service locations (consumers) in the model. There are several attributes for the consumer that will play a key role in analysis. The first is to maintain a CIS (Customer Information System) key for each service location. This would be an identifier that appears both in the GIS/EA data and the CIS/Billing system. Having this data for each customer allows load data stored in the billing system to be imported into the engineering model for analysis. This identifier will need to be unique for each customer in the model and typically is locked to a specific location, meaning if someone moves out of a house and someone else moves in, this identifier does not change.

Meter number is another attribute that can be useful. The meter number would be specific to the meter installed at the service location. It is important to store this as a separate number from the service location as meters may change at a location. The meter number may be used in the engineering analysis software to connect to AMR systems. Depending on the engineering analysis and AMR systems in place, data can be read directly from the AMR system into the model for analysis. This data includes load, blink, signal strength, and outage information.

The service wire that is the parent element of a service location is very important to maintain and store in the GIS. To run load flows with the data on the service location, it must be connected to the system. Maintaining a line section identifier for each service location makes it easier to ensure that service locations are connected correctly in the model.

Service locations do not necessarily have to represent only consumers who receive bills on the system. Other electrical elements on the system such as security lights or traffic signals may want to be considered for modeling. The reason for this is these elements consume power on the system. When performing load allocation procedures and power flow studies, the load is typically distributed on the system based on a measured demand value at a substation. This value is inclusive of all loads and losses downline from the substation, so if loads downline are not included in the circuit model, the load will still exist on the system, it will just be allocated to other consumers instead of the correct geographical location.

Once the data are collected and transferred into an engineering analysis system, analysis can be performed to determine the actual calculations on the secondary system. With these results, proper changes can be made to existing components, or new philosophies can be developed for building secondary systems in the future to improve system losses and make the entire system more efficient.

IV. Importance of a Detailed Model for Smart Grid and Real-Time Analysis

One of the many aspects of the “Smart Grid” discussion is the place of traditional distribution system modeling and analysis. Is it possible to simulate the distribution system and its ever-changing load configuration in real time using an automated cycle of continuous calculations so that the current and voltage can be known at all points 12 to 24 times a day? This is a good question that needs considerable exploration to answer, and as with most complex questions, the answers will almost certainly be complex. However, it takes only a little imagination for those who engineer and operate electrical distribution systems to see the advantages of real-time analysis and simulation. The ability to simulate the system even 12 times a day makes it possible to move from a static view to a dynamic view of the electrical system. Many believe that this ability is key to any actual smart operation of the system and that the demands of distributed generation and need for more efficient operation can only be met with real-time analysis and simulation of the distribution system.

Given that real-time simulation is at least advantageous to the ability to do “Smart Grid,” if not absolutely necessary, power system modeling companies need to evaluate what would be required. It is readily apparent that the accuracy of real-time analysis will be limited if we do not model the entire system in detail to the meter. Any attempt to accomplish real-time analysis assumes that data from Advanced Metering Infrastructure will be necessary. While it will be possible to realize some benefits from a real-time simulation of the system ending at the transformer primary, to accomplish full benefits will require modeling to as many customer meters as possible. This means accurately modeling in addition to three-phase transformers and secondary, the much more numerous center-tapped 120/240-V transformers and three wire secondaries. Real-time simulation will require kW demand data from as many meters as possible and voltage readings from strategic meters, if not from all meters possible. If the transformer and secondary are not accurately modeled, these voltage readings will be limited in value because the transformer and secondary drops will be unknown or estimated. If the model is accurate from substation to meter, then these known voltage readings can be used to validate results and adjust load flows as necessary to reach a desired tolerance from known voltage readings. Without accurate secondary data, we will be required to install transformer/meter installations at strategic locations so that we can know enough voltage points to validate accuracy and adjust to assure accuracy.

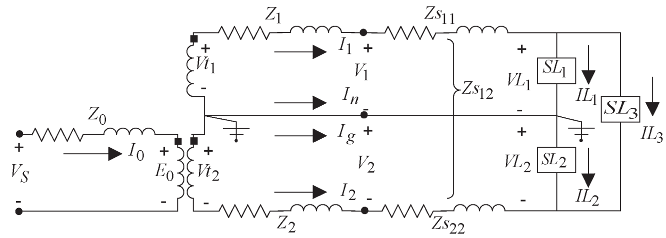

Fig. 1. Center-tapped single-phase transformer.

Given accurate secondary models, real-time load flows can be used to calculate secondary losses such that our ability to locate and replace high-loss exceptions and to eliminate over build practices is greatly enhanced. If we are confident that we will locate the exceptions, we will change to smaller transformers and secondaries in the many cases where smaller will work most of the time and thus reduce plant expansion cost. If we are confident that we can locate the exceptions, we will find and replace those transformers and secondaries that are overloaded before they can cost us unnecessary losses and/or equipment failures. Confidence in our ability to accurately simulate means that we can eliminate overbuilds practiced to avoid low-voltage installations and that we can locate and replace low-voltage installation exceptions where and when they actually exist. Will these engineering and operating advantages justify the additional expense and effort required to accurately model transformers and secondaries? Time is needed to provide this answer, but it does not take much imagination from those of us who engineer and operate the system to see the possibility that they will.

V. Center-Tapped Transformer Model

The model of a center-tapped single-phase transformer connected line-to-ground serving two 120-V loads and one 240-V load through a triplex cable is shown in Fig. 1.

The model developed for the center-tapped transformer consists of an equation for the "forward sweep" and an equation for the "backward sweep" used in the iterative routine. For the forward sweep, the general matrix equation is [1]

$$[V_{12}] = [A_t] \cdot [V_{ss}] - [B_t] \cdot [I_{12}]$$

where

$$[V_{12}] = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad [V_{ss}] = \begin{bmatrix} V_s \\ V_s \end{bmatrix} \quad [I_{12}] = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[A_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[B_t] = \begin{bmatrix} Z_1 + \frac{1}{n_t^2} \cdot Z_0 & -\frac{1}{n_t^2} \cdot Z_0 \\ \frac{1}{n_t^2} \cdot Z_0 & -\left(Z_2 + \frac{1}{n_t^2} \cdot Z_0\right) \end{bmatrix}. \tag{1}$$

In (1), the impedances must be converted to ohms relative to the respective sides of the transformer.

In Fig. 1 define

$$n_t = \frac{\text{High Side Rated Voltage}}{\text{Low Side-Half Winding Rated Voltage}}.$$

The matrix equation for the backward sweep is

$$[I_{00}] = [d_t] \cdot [I_{12}]$$

where

$$[I_{00}] = \begin{bmatrix} I_0 \\ I_0 \end{bmatrix} \quad [d_t] = \frac{1}{n_t} \cdot \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}.$$

VI. Calculating Transformer X/R

Transformers nameplate information will list the base kVA, percent impedance ($\%Z$), and voltage ratings, in addition to providing other data. The $\%Z$ can be thought of as the through impedance and is obtained from the resistance and reactance of both the primary and secondary windings. This impedance is labeled in percent on the transformer rated kVA and voltage [2].

Many times, if the percentage of resistance and reactance is unknown for medium- and high-voltage system scenarios, such as with larger distribution substation transformers over 5000 kVA, 100% of the impedance is assumed to be reactance, and any resistance is often neglected [3].

However an attempt at greater accuracy is mandatory when modeling low-voltage distribution or service transformers which must have the resistance component included for accurate fault current, load flow, arc flash, or loss calculations. Even though the majority of the impedance may still be the reactance, which varies based upon size, materials used, and manufacturing date and process, the resistive component will most definitely have an effect on the calculations particularly when considering how high residential power factors may be.

The total system X/R from the generating station to the high-side of the distribution transformer will also have a large bearing on how the transformer X/R assumptions may influence the results. For example, a long, radial medium voltage distribution line constructed from high-resistance 8A copper conductor may have a very low X/R, since the resistive component of the total impedance is much more noticeable. Compare this to a shorter line with a lower resistance 4/0 ACSR which may have a high X/R, particularly if the physical spacing between conductors is large. It is important to also keep in mind that the X component of the distribution line is very dependent upon the mutual coupling between conductors. The spacing between conductors has a large effect on this value. Furthermore, the geometric mean radius is a variable that influences the reactance component as well as the self-impedance value of the conductors themselves.

For purposes of this paper, the sensitivity analyses illustrated later assume the system upline of the distribution transformers to be zero impedance, otherwise known as infinite high-side or infinite source. As a result, the only impedance modeled in the examples is the distribution transformer and secondary wire to simplify the conclusions illustrated later.

If the full-load (winding or copper) and no-load (core) loss data are available, it is possible to calculate the transformer X/R. Typically, however, this loss data is not known particularly for older transformers already installed on the system, so reasonable assumptions may have to be made. These assumptions will be tested in the sensitivity analysis to determine how much influence they may have on calculated transformer and secondary voltages.

Transformers have both primary (R_p and X_p) and secondary (R_s and X_s) winding impedances as well as a core impedance. Since winding losses are only a function of resistance, the winding, or copper, power loss will be [4]

$$P_{\mathrm{CU}} = I^2 R = I^2 R_p + I^2 R_s$$
$$P_{\mathrm{Loss\,total}} = P_{\mathrm{CU}} + P_{\mathrm{core}}. \tag{2}$$

To find X and R, the total power loss must be known, and either the copper or core losses must be known as well to satisfy (2). Since the winding losses are a function of the transformer resistance, this value can be calculated by using the above formulas as well as the transformer nameplate percent impedance. Following is an example for illustration.

Three-phase kVA	300 kVA
Voltage	12470/7200–208/120 V
Nameplate $\%Z$	4.0%
Total losses	4500 W
Load losses	4000 W

$$I = \frac{kVA}{\sqrt{3} * kV} = \frac{300}{\sqrt{3} * 0.208} = 830 \text{ Amps}$$
$$P = I^2 * R.$$

Since three phase, then

$$P = 3 * I^2 * R$$
$$P = 3 * 830^2 * R = 4000 \text{ W}$$
$$R = 0.00194 \text{ ohms}$$
$$\%R = \frac{R * kVA}{10 * kV^2} = \frac{0.00194 * 300}{10 * 0.208^2} = 1.345\%.$$

Since

$$\%Z = \sqrt{\%R^2 + \%X^2}$$
$$\%X = \sqrt{4.0^2 - 1.345^2} = 3.77\%.$$

Therefore,

$$\frac{X}{R} = \frac{3.77}{1.345} = 2.8.$$

There are countless industry references, standards, and utility surveys of extreme value that derive and provide typical impedance values and X/R ranges for distribution transformers. Generally, these may contain values relevant to noncenter-tapped two-winding single-phase transformers with 120-, 240-, or 480-V secondary ratings, or three-phase padmounted transformers, but some contain center-tapped transformer values. A few of these found to be useful for engineering analysis in general as well as for discussion and example purposes of this paper are listed as follows.

- ANSI standard C57.12.38-2009.
- RUS Bulletin 160-3 Service to Induction Motors, pg. 39, Table E.
- GE Short Circuit Calculations for Industrial and Commercial Power Systems, Appendix Table 11.
- Cutler-Hammer Consulting Application Guide Reference Data.

Consideration also needs to be given to the Department of Energy Distribution Transformer Efficiency Standards since distribution transformers must meet minimum efficiency levels effective January, 2010. Since both core and winding losses are part of the efficiency equation, utility personnel must make note of this and ensure that new services added to engineering models include revisions to equipment data associated with the correct specifications for new transformers [5].

VII. Center-Tapped Transformer X and R

As shown in Fig. 1, schematic for the center-tapped transformer model, distribution transformers, often have a single primary winding and two secondary windings of the same voltage. For example, a common transformer rating may be 25 kVA, 7200/12470Y to 120/240 V, with one 7200-V primary winding and two 120-V windings.

Distribution transformers such as these may be connected as three-phase banks to provide service to a higher percentage of three-phase 240-V loads and a lesser percentage of 120/240 V single-phase loads. With this connection, two transformers are often used as two-winding transformers and a third as a three-winding transformer. This allows using the midpoint of the two secondary windings of the three-winding transformer to be grounded. This bank could then serve 240-V three-phase loads as well as both 120- and 240-V single-phase loads.

Because the three-winding transformer may be used also as a two-winding transformer to provide 240-V single-phase service, it is important to be able to obtain impedance data for this or any other possible type of connection. In addition, the transformer may have interlaced or noninterlaced windings

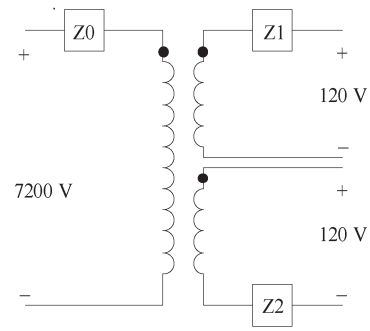

Fig. 2. Typical three-winding transformer.

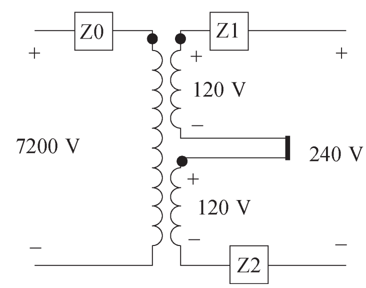

Fig. 3. Two-winding connection.

which results in different impedances between the primary and secondary windings of each.

When referencing transformer impedance data given for two-winding transformers, the impedances for each of the three windings for a center-tapped transformer must be calculated to increase the accuracy for the engineering studies.

Fig. 2 shows a three-winding, or center-tapped, distribution transformer. A common rating is 25 kVA, and as shown, a 7200-V primary winding and two 120-V secondary windings.

Fig. 3 shows the secondary windings connected together to form a single 240-V winding. As shown below, the three-winding transformer has been connected as a two-winding transformer. Many references supply impedance data for this type of connection.

Table I is a reference from the General Electric Application Information for Short-Circuit Current Calculations for Industrial and Commercial Power Systems and shows the impedance data for a two-winding transformer connection [6].

Since the total impedance of the transformer $(R_A + jX_A)$ is known, this can then be broken into three parts, one for the primary (Z_0) and each secondary winding (Z_1 and Z_2) as shown in Fig. 2. For interlaced design, the three impedances are given by [1]

$$\begin{bmatrix} Z_0 \\ Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 0.5 \cdot R_A + j0.8 \cdot X_A \\ R_A + j0.4 \cdot X_A \\ R_A + j0.4 \cdot X_A \end{bmatrix} \text{ per unit.}$$

For a noninterlaced design, the three impedances are given by [1]

$$\begin{bmatrix} Z_0 \\ Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 0.25 \cdot R_A - j0.6 \cdot X_A \\ 1.5 \cdot R_A + j3.3 \cdot X_A \\ 1.5 \cdot R_A + j3.1 \cdot X_A \end{bmatrix} \text{ per unit.}$$

TABLE I
TYPICAL TWO-WINDING TRANSFORMER IMPEDANCES
12470Y/7200 TO 120/240 AND 240/480 V

kVA	%R_A	%X_A	%Z_A	X_A / R_A
120/240				
10	2.5	0.9	2.7	0.4
15	2.1	1.6	2.6	0.8
25	1.6	2.0	2.6	1.3
37.5	1.6	2.5	3.0	1.6
50	1.2	2.0	2.3	1.7
75	1.0	2.0	2.2	2.0
100	0.8	2.2	2.3	2.8
167	1.0	2.0	2.2	2.0
240/480				
10	2.5	0.8	2.6	0.3
15	2.1	1.5	2.6	0.7
25	1.6	1.9	2.5	1.2
37.5	1.0	1.5	1.8	1.5
50	1.1	1.8	2.1	1.6
75	0.9	1.8	2.0	2.0
100	0.7	1.8	1.9	2.6
167	0.9	1.7	1.9	1.9

*All impedances are in per-unit based upon the transformer rating.

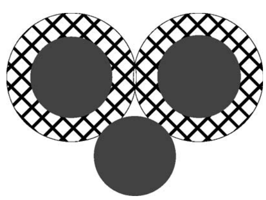

Fig. 4. Triplex cable.

Note that with the noninterlaced, the impedance between the primary and one 120-V secondary is different from the impedance between the primary and the other 120-V secondary winding. For interlaced, the impedances between the primary and both secondary windings are equal.

VIII. TRIPLEX CABLE MODEL

Triplex cable is used with single unit center-tapped transformers when providing common residential service. The cable consists of two insulated conductors and one bare conductor as shown in Fig. 4.

The triplex cable can be modeled with 2×2 matrices [1]. With reference to Fig. 1, the forward sweep matrix equation for the triplex cable is

$$[VL_{12}] = [A_s] \cdot [V_{12}] - [B_s] \cdot [I_{12}].$$

The triplex impedances are computed by applying Carson's Equations to compute the 3×3 primitive impedance matrix [1]

$$[zp] = \begin{bmatrix} [zp_{ij}] & [zp_{in}] \\ [zp_{nj}] & [zp_{nn}] \end{bmatrix} \ \Omega/\text{mile}.$$

The Kron reduction is used to create the 2×2 impedance matrix as shown by

$$[z_{\text{sec}}] = \begin{bmatrix} zs_{11} & zs_{12} \\ zs_{21} & zs_{22} \end{bmatrix} \ \Omega/\text{mile}.$$

In the process of applying the Kron reduction, an equation is developed that allows for the computation of the current flowing in the neutral wire as a function of the two secondary line currents

$$I_n = [tn] \cdot [I_{12}]$$

where

$$[tn] = -[zp_{nn}]^{-1} \cdot [zp_{nj}].$$

Kirchhoff's current law must apply so that the current flowing in the ground path is determined by

$$I_g = -(I_1 + I_2 + I_n).$$

IX. SENSITIVITY ANALYSIS

Due to the inconsistencies that can occur between the distribution equipment installed on the electrical system and what exists in the GIS or engineering software secondary service model, a few sensitivity analyses were performed to observe the effects on the voltage profiles of the center-tapped transformer and secondary conductor. As discussed in the GIS-EA Section III of the report, the largest number and variety of distribution equipment types will be the distribution transformers. Even if there is a size and type range stocked in the warehouse such that the nameplate ratings may be readily accessible, for older units installed on the system, the paperwork or reference data for these may be difficult to locate.

An example related to varying the percentage of load unbalance has also been investigated to determine voltage impact delivered to the end-use customer. Even if it is felt that the engineering model is 100% accurate from an equipment specification standpoint, the dynamic nature of the load between multiple consumers served from the same transformer may be one of the most difficult variables to determine. High-resolution data sampling for measuring noncoincident load curve data needed to ensure load modeling accuracy is not readily available to most distribution system planners. [7]

Milsoft's WindMil engineering software was used to model the systems used in the sensitivity analyses [8]. The algorithms for the aforementioned derivations for center-tapped transformers and the X and R for each of the three windings, along with the secondary triplex cable have been implemented into the WindMil software algorithms. Any available power system modeling software can be used if it is truly capable of center-trapped transformers and secondary conductor calculations.

Sensitivity analyses were performed on a few possible variables potentially unknown in real-world applications such as:

- Distribution transformers percent impedance.
- Distribution transformers X/R.
- Load balance on 120 V secondary windings.

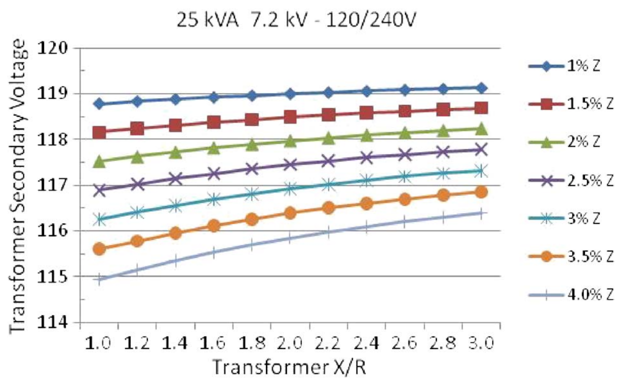

Fig. 5. Secondary voltage levels with no triplex.

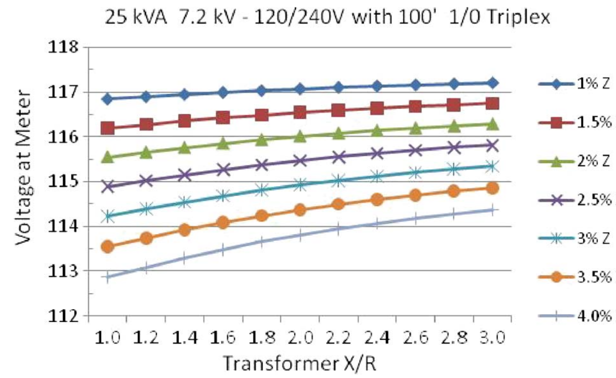

Fig. 6. Secondary voltage levels with 1/0 triplex secondary.

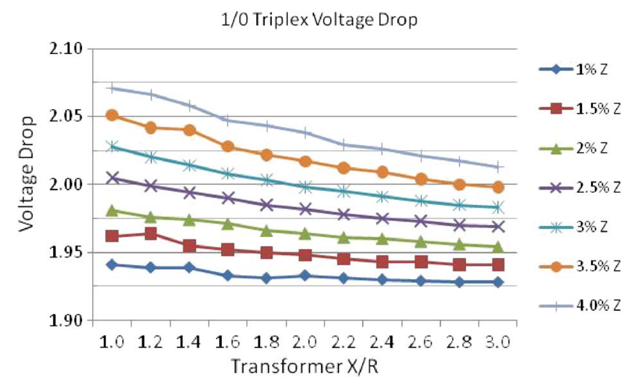

Fig. 7. Secondary conductor voltage drop.

The following system was modeled in the Milsoft WindMil engineering software.

System Attributes

- Distribution transformer: 25 kVA 7.2 kV–120/240 V.
- 120-V Half-Winding 1:5 kW + j1.6434 kVAR, 95% PF.
- 120-V Half-Winding 2:5 kW + j1.6434 kVAR, 95% PF.
- 240-V Full-Winding:15 kW + j7.3 kVAR, 90% PF.
- All loads constant PQ (Constant kVA) load mix.

To simplify the results for Examples 1 and 2, the loading on each 120-V half-winding was set to be equal such that the voltages across each 120 V winding would also be equal. The power factor of the 240-V loads are set to a slightly lower power factor of 90% which is considered reasonable for 240-V residential load types. Furthermore, this kVAR demand factored in helps demonstrate the effects of the reactive component of the transformer impedance on the delivered voltages.

The examples are set with an infinite source bus at the primary voltage terminals of the distribution transformer and held at a delivered voltage of 7200 V. As a result, the only impedance modeled in the examples is the distribution transformer and secondary wire for simplification.

A. Example 1: 25 kVA With No Triplex

Fig. 5 illustrates the voltage delivered at the secondary terminals of the 25 kVA center-tapped transformer. The percent impedance was varied from one percent to four percent while the X/R ranged from 1.0 to 3.0.

B. Example 2: 25 kVA With 1/0 AA Triplex

The same system as in Example 1 was modeled, but 100 ft of 1/0 AA Triplex with 1/0 ACSR bare neutral secondary cable was added to evaluate the voltage profile effects on the conductor as the transformer impedance values were varied. Fig. 6 illustrates the delivered voltage variations to the customer meter through both the transformer and secondary 1/0 Triplex.

For the transformer impedances modeled, the voltage profiles are similar between the cases with and without the secondary 1/0 triplex. Note that essentially, the voltage drop through the secondary conductor is 2.0 $+/-$ 0.10 V when comparing the voltage delivered difference between Figs. 5 and 6. Fig. 7 shows this voltage change for each transformer impedance scenario. Both 120-V and 240-V loads modeled have been set as constant PQ or constant kVA, so as the voltage drop through the transformer increases, the load current through the triplex must increase as well so that the total kVA, or PQ, of the load remains constant. In other words, there is simply an inverse relationship between voltage and current to maintain the constant kVA load.

Even though the small $+/-0.10$-V variance on the secondary conductor can be considered minimal with this example, it simply signifies that load mix must also be considered as one of the variables that need to be specified when modeling secondary systems and loads. For lower power factor loads with a higher reactance component of the total impedance, the $+/-0.10$-V variations would become more severe.

Since the triplex was modeled as a bundled conductor, the geometric mean distance between the phase conductor and neutral is roughly 0.5 in. This tight configuration effectively results in a significantly greater resistance component compared to the reactance. Furthermore, note as the X/R decreases for each of the impedances tested, the voltage drop across the triplex increases inversely as a result of the load being modeled with such a high power factor, thus mostly active load rather than reactive.

It must be pointed out that the total impedance contribution from the transformer and secondary triplex must be taken into account in the preceding examples. Of course, smaller triplex or a longer run of secondary, or even a triplex with reduced neutral will weaken the case of manipulating the transformer impedance tolerances since that would dilute the total system impedance.

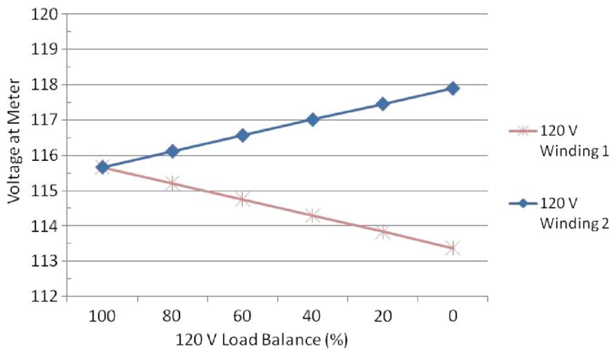

Fig. 8. Voltage from half-winding load imbalance.

C. Example 3: Unbalanced 120 V Loads

One of the most significant challenges with modeling secondary systems that accurately depict real-world situations is not in the details of the properties or specifications of the equipment, but in the load type, demand, power factor, and balance. The load balance is particularly important when evaluating center-tapped transformers due to each 120-V secondary winding and resultant voltage drop through each 120 V phase wire of the secondary triplex.

Assumptions can be made in regard to the percentage of loads on a center-tapped transformer that are 120 V compared to 240 V rated (clothes dryers, ovens, furnaces, or air-conditioning units), but when accounting for diversity and peak coincidence factors, these loads on each center-tap phase vary continuously. Since typically residential meters may only record the total kWh usage, not separate usage of the 120 V and 240 V loads, this makes the modeling accuracy even more challenging.

As noted in Table I, common 25-kVA center-tapped transformer impedances may be in the range of 2.6% with an X/R of 1.3. The transformer in this example has been set to 2% impedance with an X/R of 1.5 for comparison purposes with the two previous examples 1 and 2.

Fig. 8 illustrates that when varying the load balance of 10 kW of 120-V single-phase loads, a voltage deviation of almost 4.0 V, or $+/-$ 2.0 V from a completely balanced situation, results.

Some additional sensitivity variables for potential further study to realize their impact on secondary voltage systems would include secondary conductor spacing, secondary conductor with reduced neutral, load power factor, and load type (constant kVA (PQ), constant impedance, or constant current), as well as the transformer loading percentage.

X. Secondary Conductor Modeling Requirements and Characteristics

When comparing secondary service conductor to distribution transformers, there are many less data accuracy variables that could potentially be unknown for the secondary cable, therefore sensitivity analyses for secondary wire were not completed. However, to still be able to completely and accurately represent the physical and electrical properties of the conductor and construction, the following values must be identified in the GIS and secondary conductor equipment database.

1) Conductor resistance at operating temperature.
2) Geometric mean radius.
3) Conductor diameter.
4) Construction spacing between phases and neutral.
5) Earth resistivity.
6) Secondary service conductor length.

A secondary service guide or schedule created by the utility according to their predefined guidelines will almost always be used when deciding upon the appropriate conductor size for the application. This is typically dependent upon the loading dynamics, transformer size installed, and length of secondary conductor needed. With this, the utility may stock up to three or four different wire sizes of triplex as well as quadriplex, among others, and will therefore know the conductor specifications (listed in numbers one through three above) as these can be obtained from the manufacturer.

This leaves the last three variables from the list above needing values determined for an accurate depiction of the installation. First, conductor spacing with any bundled configuration can be modeled in a "random" spacing configuration or otherwise a relatively small distance between the centers of each. Commonly, 0.5 in up to 3.0 in may be used with distribution secondary; obviously, this distance is dependent upon the conductor diameter. Either way, the mutual inductive reactance and shunt admittance of bundled secondary can be considered minimal relative to the self-inductance, or resistance, of the conductor alone. Second, the earth resistivity will affect the percentage of unbalanced load current that flows through the earth versus the neutral conductor. A value of 100 ohm-m is common for earth resistivity. The last variable above, secondary service length, will have the most significant impact on the power analysis results since a large percentage of the low-voltage resistive component of the total impedance is attributable to this cable.

XI. Summary

Software evolution has clearly made it not only possible but also much simpler to handle the vast amount of data needed for accurate modeling of secondary voltages of distribution systems. Implementing real-time state estimation algorithms into engineering software applications will furthermore heavily rely on this precision.

One of the most important pieces of information needed in power flow, fault current analyses, and other power system simulations is the transformer impedance. When considering the number of low voltage distribution transformers installed, the transformer impedance and X/R values may vary significantly, and unfortunately, this information may also not be readily available, challenging the utility to determine the most appropriate and easiest way to handle this situation.

Examples modeled with the WindMil engineering software indicated the $\%Z$ has a much more significant impact compared to the X/R on the power flow results for typical residential load types with a higher power factor. The results are very much directly related to the load power factor. Therefore, one perspective is that the utility may want to maintain an equipment database that contains similarly sized center-tapped transformers with different percent impedances, but just one X/R for each size, or min/max impedance values in which one or the other would be used to provide conservative results based on the type of study conducted, such as using the minimum value for fault current analysis and the maximum value for load flows. This is even more important considering that most utility systems are vastly residential in nature and contain a very high percentage of center-tapped transformers.

Load balance accuracy on each 120-V winding also can drastically influence the voltage delivered to the customer and may just be the sensitivity variable that makes or breaks whether the utility meets ANSI voltage criteria standards, or any other operations or planning criterion or guidelines within the utility [9].

Although the transformer and loading values that were varied in the examples may be more extreme than what would be the case for distribution transformer applications found on a distribution system, these exercises definitely show that there can be voltage drop disparities between engineering software packages secondary service model results and what may actually be occurring on the system.

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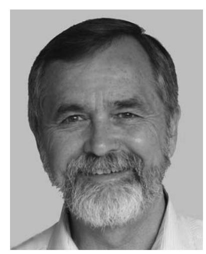

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